Find My Number

I'm thinking of a 10 digit number where...

- Each digit from 0 to 9 is used exactly once.
- The first *n* digits are divisible by *n*.

What could my number be?

This means, if my special number is 1234567890, then
1 (the first digit) is divisible by 1
12 (the first two digits) is divisible by 2
123 (the first three digits) is divisible by 3
1234566 (the first 6 digits) is divisible by 6, etc.
(Incidentally, 1,234,567,890 breaks at the fourth digit)

A number is divisible by another number if you get a whole number when you divide them.
For example, is 15 divisible by 4? No because 15/4 = 3.75, not a whole number.
But, is 15 divisible by 5? Yes, because 15/5 = 3, a whole number.

There are also some tricks that make it much easier to tell, like *anything that's divisible by 10 has a zero on the end*. Here are some helpful rules (great if you don't have a calculator, but also great for finding helpful patterns along the way):

Divisibility Rules:

1. **All, of course**
2. **Even last digit**
3. **Sum of digits divisible by 3 (repeat if needed)**
4. **Last 2 digits divisible by 4**
5. **Last digit 0 or 5**
6. **Even and divisible by 3**
7. ~~**Double the last digit and subtract it from the remaining digits, the result is divisible by 7**~~
   Just use a calculator on this one.
8. **Last 3 digits divisible by 8 (halve 3 times)**
9. **Sum of digits divisible by 9 (repeat if needed)**
10. **Last digit 0**

Find The 10 Digit Number Where N Digits Are Divisible By N – Sunday Puzzle

Posted April 10, 2016 By Presh Talwalkar

On March 14 (Pi Day), Pizza Hut parterned with the famous mathematician John Conway and ran an interesting contest. They posted 3 math puzzles, and the first person to answer a problem won 3.14 years of free pizza (valued at $1,600).

The first problem is a classic in recreational mathematics, which I will rephrase as follows.

"Find a 10 digit number that uses each of the digits 0 to 9 exactly once and where the number formed by the first *n* digits of the number is divisible by *n*."

You should read the digits of the number from left to right. For example, in the number *abcd*, you need the number *a* to be divisible by 1, the number *ab* to be divisible by 2, the number *abc* to be divisible by 3, and the entire number *abcd* to be divisible by 4.

There are terms for this kind of number. A number where its first *n* digits are divisible by *n* is known as polydivisible number. A decimal number that uses all the digits 0 to 9 at least once is known as a pandigital number.

In this terminology, the puzzle is to find a 10 digit pandigital number that is also polydivisible.

You could check all the possible combinations. But the fun part is this problem can be solved by following logical steps and eliminating possibilities.

Can you figure it out?

ONE WAY TO SOLVE THIS:

There are multiple methods to solve this problem. I will offer a solution that makes sense to me.

I will write the 10 digit number using alphabet letters:

abcdefghij

How many possible numbers are there to check? The first digit could be any of 10 numbers, the second digit could be any of the 9 remaining numbers, the third digit could be any of the 8 remaining numbers, and so on. In all there would be 10×9×8×7×6×5×4×3×2×1 = 10! = 3,628,800 possibilities. This is a lot of numbers to check, so let's reduce the search.

To start, we need the entire 10 digit number to be divisible by 10. This means the last digit has to be 0. So we have $j = 0$, and the number looks like:

*abcdefghi*0

This is a small step, but it reduces the search by a factor of 10, so there are 362,880 possibilities to check.

Next we need the first 5 digits to be divisible by 5. A number is divisible by 5 if and only if its last digit is 0 or 5. Since have already used 0, that means the fifth digit has to be 5, so $e = 5$.

*abcd*5*fghi*0

This is another small step, but it reduces the search by a factor of 9, so there are 40,320 possibilities to check. We can use some more logic to reduce the search further.

The digits *b*, *d*, *f* and *h* have to be even because *ab* has to be divisible by 2, *abcd* has to be divisible by 4, *abcdef* has to be divisible by 6, and *abcd*5*efgh* has to be divisible by 8. This means the digits *b*, *d*, *f* and *h* are the available even digits 2, 4, 6, and 8.

This also means the other digits *a*, *c*, *g* and *i* come from the remaining numbers 1, 3, 5, 7, 9.

The digits in the even spots are even and the digits in the odd spots are odd.

The 4 even digits can be arranged in 4x3x2x1 = 24 possibilities, and the 5 odd digits can be arranged in 5x4x3x2x1 = 120 possibilities. That means there are 24×120 = 2,880 possibilities to check. We've reduced the search from over 3 million to less than 3 thousand using simple divisibility rules!

To advance further using logic, we need to use more divisibility rules. The number *abcd* has to be divisible by 4, which happens if its last two digits are.

Since *c* is odd, but not 5, and *d* is even but not 0, we can work out the possibilities for *cd*:

12
16
32
36
72

76
92
96

This means the digit *d* has to be either 2 or 6.

Let's look at the digits *def*. We already know *d* is 2 or 6 and *e* is 5. We also know *f* has to be even but not 0, so it has to be 4, 6 or 8. So we have the following possibilities:

254
256
258
652
654
658

Now we deduce another fact about *def*. We know the number *abcdef* is divisible by 6, and therefore it must also be divisible by 3. Since *abc* is divisible by 3, we also have *def* must be divisible by 3, so that the sum of its digits are a multiple of 3. This leaves only 2 possibilities from the above:

258
654

So our number looks like the following:

*abc*258*ghi*0
*abc*654*ghi*0

Now we deduce something about *fgh*. We know the number *abcdefgh* is divisible by 8, which means its last 3 digits *fgh* have to be divisible by 8.

If *f* = 8, here are the possibilities.

800
808
816
824
832
840
848
856
864
872
880
888
896

We also need *g* to be odd but not 5, and we need *h* to be even but not 0 or any other number used. This leaves only the numbers 816, 832, 872, and 896. But we have to exclude 832 and 872 because when *f* = 8 we also have *d* = 2 (we have used the 2 digit already).

So if the number is *abc*258*ghi*0, then the possibilities could be:

abc25816i0
abc25896i0

Because b is even, it must be 4 in these cases because all the other even numbers are used:

a4c25816i0
a4c25872i0

If $f = 4$, here are the possibilities.

400
408
416
424
432
440
448
456
464
472
480
488
496

We also need g to be odd but not 5, and we need h to be even but not 0 or any other number used. This leaves only the numbers 416, 432, 472, and 496. But we have to exclude 416 and 496 because when $f = 4$ we also have $d = 6$ (we have used the 6 digit already).

If the number is abc654ghi0, then the possibilities could be:

abc65432i0
abc65472i0

Because b is even, it must be 8 in these cases because all the other even numbers are used:

a8c65432i0
a8c65472i0

To sum up using the divisibilty by 8 rule, here are the only possible numbers:

a4c25816i0
a4c25896i0
a8c65432i0
a8c65472i0

Now the first 3 digits abc have to be divisible by 3, and the digits a and c cannot be any of the digits already used. Once we know those digits, the last digit i is determined too.

Now we can start guessing. Consider the case a4c25816i0. If the first digit is 3, then the third digit would have to be 7 or 9. But neither would make for the 3 digits to have a sum divisible by 3, so the first digit cannot be 3. Similarly, the first digit cannot be 7 or 9, so this entire case does not work.

Consider the case *a*4*c*25896*i*0. If the first digit is 1, then the third digit would have to be 7, meaning the ninth digit has to be 3. We could also exchange the 7 and 1 for another possibility:

1472589630
7412589630

We can similarly work out possibilities for the other cases:

*a*8*c*65432*i*0
*a*8*c*65472*i*0

In the first case *a*8*c*65432*i*0, the numbers 1, 7 and 9 can be arranged in 4 possibly valid ways:

1896543270
9816543270
7896543210
9876543210

In the second case *a*8*c*65472*i*0, the numbers 1, 3 and 9 can be arranged in 4 possibly valid ways:

1836547290
3816547290
1896547230
9816547230

So in all, we have 10 possible numbers to check:

1472589630
7412589630
1896543270
9816543270
7896543210
9876543210
1836547290
3816547290
1896547230
9816547230

Now we need to check that the first 7 digits are divisible by 7 (you could do this by dividing, or you could do the divisibility by 7 graph).

Only one of the numbers is:

3,816,547,290

And that is the answer!

Source: https://mindyourdecisions.com/blog/2016/04/10/find-the-10-digit-number-where-n-digits-are-divisible-by-n-sunday-puzzle/

Divisibility Rules:

1. All, of course
2. Even last digit
3. Sum of digits divisible by 3 (repeat if needed)
4. Last 2 digits divisible by 4
5. Last digit 0 or 5
6. Even and divisible by 3
7. Double the last digit and subtract it from the remaining digits, the result is divisible by 7 (or use chart below)
8. Last 3 digits divisible by 8 (halve 3 times)
9. Sum of digits divisible by 9 (repeat if needed)
10. Last digit 0